Domain Decomposition and Its Applications in Time-Stepped Nonlinear Finite Element Analysis

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Abstract —This paper presents a domain decomposition method (DDM) for time-stepped nonlinear finite element analysis (FEA) and its applications in electrical machines. This method, different from the classical Schwarz method, has no overlapping region between two sub-domains and, thus, does not require an iteration process to reach the accurate solution for linear problems. Due to the absence of the overlapping region, each sub-domain can belong to different physical domains, as an example, one being circuit sub-domain while others being FEA sub-domains. This feature not only makes it possible to support parallel computing, but also simplifies the formulation and implementation. Since the proposed DDM itself does not involve iteration, this method has no side effects on Newton-Raphson iteration in the case of nonlinear problems. The proposed DDM has been applied to transient FEA-circuit coupled problems and motion problems with the stationary and the rotating parts as separate sub-domains.

I. INTRODUCTION

Time-stepped finite element analysis (FEA) is a useful tool for the performance prediction of electrical machines. The major drawback of time-stepped FEA is computation time cost. A typical simulation of a single operating point may require the solution of thousands of time steps [1]. To accelerate the process of the time-stepped electromagnetic field computation, the domain decomposition method (DDM) has been used to accelerate the computation [1]-[6]. In [2], the Jacobian matrix is partitioned into linear and nonlinear blocks, thereby allowing the relatively rapid generation of an efficient multiplicative preconditioner for the conjugate gradient (CG) iteration. The multi-slice FEA [4] is used to consider the skew effects in induction machines; each slice is taken as a sub-domain and can be solved by parallel computing. Recently, the DDM is utilized in rotating machines with the stationary part and the rotating part taken as two sub-domains [6], and Krylov subspace iteration is used to speed up the simulation.

The classical DDM decomposes the whole domain into several sub-domains with overlap regions between any two adjacent sub-domains [3], [5], [7]. The solution in the whole domain is reached by solving each sub-domain iteratively.

This paper presents an alternative DDM without overlapping regions between two adjacent sub-domains. LU matrix factorizations are performed by parallel computing for the stiffness matrixes in all sub-domains, and unknowns on all interior boundaries and all sub-domains are computed successively. Due to the absence of the overlapping region, each sub-domain can belong to different physical domains, as an example, one being circuit sub-domain while others being FEA sub-domains. This feature simplifies the formulation and implementation. Since the proposed DDM itself does not involve iteration, this method has no side effects on Newton-Raphson (NR) iteration in the case of nonlinear problems. The proposed DDM has been applied to transient FEA-circuit coupled problems and motion problems with the stationary and the rotating parts as separate sub-domains.

II. PRESENTED DOMAIN DECOMPOSITION METHOD

In the presented DDM, the whole solution domain is decomposed into several sub-domains without overlapping regions, as shown in Fig. 1.

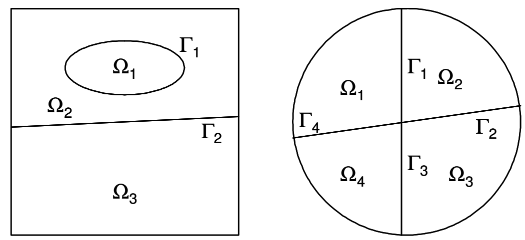

Fig. 1. DDM without overlapping regions

In general, if the whole solution domain is divided into n sub-domains, then the equation in the whole domain is

$$\begin{bmatrix} \mathbf{A}_{11} & 0 & \cdots & 0 & \mathbf{A}_{1\Gamma} \\ 0 & \mathbf{A}_{22} & \cdots & 0 & \mathbf{A}_{2\Gamma} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \mathbf{A}_{nn} & \mathbf{A}_{n\Gamma} \\ \mathbf{A}_{\Gamma 1} & \mathbf{A}_{\Gamma 2} & \cdots & \mathbf{A}_{\Gamma n} & \mathbf{A}_{\Gamma\Gamma} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \cdots \\ \mathbf{X}_n \\ \mathbf{X}_\Gamma \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \cdots \\ \mathbf{Y}_n \\ \mathbf{Y}_\Gamma \end{bmatrix} \tag{1}$$

where Γ denotes the set of all interior boundaries. The solution of (1) is

$$\mathbf{X}_\Gamma = (\mathbf{A}_{\Gamma\Gamma} - \sum_{k=1}^{n} \mathbf{A}_{\Gamma k}\mathbf{B}_{k\Gamma})^{-1}(\mathbf{Y}_\Gamma - \sum_{k=1}^{n} \mathbf{A}_{\Gamma k}\mathbf{X}_k^{'}) \tag{2}$$

$$\mathbf{X}_k = \mathbf{X}_k^{'} - \mathbf{B}_{k\Gamma}\mathbf{X}_\Gamma \qquad (k = 1,2,\cdots,n) \tag{3}$$

where

$$\begin{cases} \mathbf{X}_k^{'} = \mathbf{A}_{kk}^{-1}\mathbf{Y}_k \\ \mathbf{B}_{k\Gamma} = \mathbf{A}_{kk}^{-1}\mathbf{A}_{k\Gamma} \end{cases} \quad (k = 1,2,\cdots,n)\,. \tag{4}$$

The solution sequence is first to solve (4) by parallel computing for each sub-domain; then to solve (2) for all interior boundaries; finally to obtain solutions of all sub-domains by (3). For each sub-domain, (4) can be efficiently solved in one Gaussian Elimination with multiple right-hand side vectors [$\mathbf{Y}_k$ $\mathbf{A}_{k\Gamma}$], or be solved with only one LU matrix factorization.

Different from the classical DDM, the presented DDM solves the linear equation without iteration, and the solution is exactly the same as that directly from (1). Therefore, the presented DDM has no side effects on NR iteration in nonlinear FEA.

III. APPLICATIONS

Since there are no overlapping regions in the presented DDM, the solution types, or some properties, in all sub-domains can be different from each other. This feature simplifies the formulation in each sub-domain. Some typical applications are introduced below.

A. FEA and Circuit Sub-Domains

If FEA is coupled with controlling circuit, the whole coupled equation can be divided into two sub-domains: one for FEA and the other for circuit. The interior boundary between two sub-domains stands for all independent winding currents $\mathbf{I}_w$.

The symmetric FEA equation can be expressed as

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{1w} \\ \mathbf{A}_{1w}{}^T & \mathbf{A}_{ww} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{I}_w \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{V}_w \end{bmatrix} \tag{5}$$

where $\mathbf{V}_w$, the vector for all independent winding terminal voltages, is a unknown vector. The circuit system equation can be written as

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{1w} \\ \mathbf{Z}_{w1} & \mathbf{Z}_{ww} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_w \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{V}_w \end{bmatrix}. \tag{6}$$

Cancelling right-hand side unknown vector $\mathbf{V}_w$ in (5) and (6), one obtains

$$\begin{bmatrix} \mathbf{A}_{11} & 0 & \mathbf{A}_{1w} \\ 0 & \mathbf{Z}_{11} & \mathbf{Z}_{1w} \\ \mathbf{A}_{1w}{}^T & \mathbf{Z}_{w1} & \mathbf{A}_{ww} + \mathbf{Z}_{ww} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{I}_1 \\ \mathbf{I}_w \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{V}_1 \\ 0 \end{bmatrix} \tag{6}$$

which has the format of (1) and can be solved by the presented DDM.

B. Linear and Nonlinear Sub-Domains

If the whole solution domain is divided into a linear sub-domain Ω_1 and a nonlinear sub-domain Ω_2, the equation as expressed in (1) is normally solved by the NR iteration.

During the NR iteration, the stiffness matrix in the linear sub-domain keeps unchanged, thus only one LU matrix factorization is necessary. Further, if the linear sub-domain does not involve motion, the factorized LU matrixes can be recycled during the entire transient process, which makes the computation very efficient.

C. Stand-Still and Rotating Sub-Domains

If there is rotational motion in the solution domain, one can divide the whole domain into a stand-still sub-domain for the stator and a rotating sub-domain for the rotor with the interior boundary locating at the air-gap center. The field equation with sliding interface [6] has the format of (1), and therefore, the presented DDM is applicable.

If both the stator and rotor involve nonlinear materials, one can divide the whole domain into four sub-domains: the linear and nonlinear sub-domains in both stator and rotor. In such a case, there is no motion in each linear sub-domain, and therefore, only one LU matrix factorization is necessary for the stiffness matrix in each linear sub-domain during the whole time-stepped simulation.

There are more applications of the presented DDM in FEA of rotating electrical machines, such as the application in the multislice FEA for skewed induction motors [4].

IV. EXAMPLES

The first example is for the application of the FEA and circuit sub-domains. A 900W, 24V, 136rpm, 3-phase, 22-pole, 24-slot brushless dc motor with outer rotor structure is studied. The computed and measured stator phase currents are shown in Fig. 2 and Fig. 3, respectively.

More examples will be provided in the full paper.

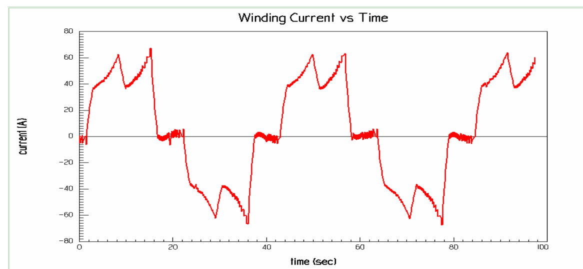

Fig. 2 Computed phase current

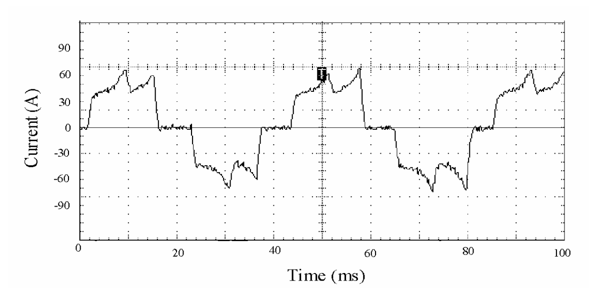

Fig. 3 Measured phase current

V. REFERENCES

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